The DSP Powerhouse

White Paper

CEVA-XC

MIMO Maximum Likelihood Detector (MLD)

Noam Dvoretzki – Senior HW Architect, CEVA

Zeev Kaplan – Senior Communication Algorithm Engineer, CEVA

March 2014, Rev 1.0

Table of Contents

1 INTRODUCTION 1

2 MIMO TECHNIQUES 2

2.1 ANTENNA CORRELATION 4

3 MIMO RECEIVERS 5

3.1 TURBO MIMO 8

4 ML RECEIVER IMPLEMENTATION 9

4.1 SUBOPTIMAL ML RECEIVERS 10

4.1.1 Breadth First Search 11

4.1.2 Depth First Search 12

5 CEVA SOLUTION 13

5.1 FEATURE SET 14

5.2 MLD ACCELERATOR BLOCK DIAGRAM 14

5.3 MLD PERFORMANCE 15

6 CONCLUSION 17

7 REFERENCES 18

Table of Figures

Figure 2-1: Tx Beam-Forming 2

Figure 2-2: STBC Throughput Peak 3

Figure 2-3: Antenna Correlation Effects 4

Figure 3-1: Diversity Order 6

Figure 3-2: Array Gain 6

Figure 3-3: Turbo MIMO Receiver 8

Figure 4-1: MIMO Symbol Tree 10

Figure 4-2: K-Best Tree Traversal 11

Figure 4-3: Fixed vs. Adaptive Complexity 12

Figure 4-4: Sphere Decoder Tree Traversal 13

Figure 5-1: MLD Accelerator Block Diagram 14

Figure 5-2: 4X4 MIMO Spatial Multiplexing Performance 15

Figure 5-3: MLD 4X4 MIMO Performance 16

Figure 5-4: MLD 2X2 MIMO Performance 16

List of Tables

Figure 2-1: Tx Beam-Forming 2

1 Introduction

Companies in today's world are constantly striving to provide solutions to the growing demand for higher data rates in mobile devices. Given that radio spectrum is limited and expensive, it is vital to discover a better way to utilize the same bandwidth, while transmitting even more data – or in other words: to improve the spectral efficiency of the channel.

MIMO (Multiple Input and Multiple Output) is one of the leading approaches for improving data rates and/or SNR (Signal to Noise Ratio). By using multiple receive and transmit antennas, MIMO can exploit the diversity of the wireless channel. This is then used to increase the spectral efficiency of the channel and improve the data rates for any given channel bandwidth.

The MIMO dimension depends on the number of antennas transmitting and receiving. In a 4X4 MIMO configuration, four transmit antennas and four receive antennas are used. This enables (under the right conditions) transmitting up to four times more data on the same channel bandwidth.

On the one hand, a simple MIMO receiver is based on a linear receiver algorithm which is easy to implement but cannot fully exploit the MIMO benefits. On the other hand, an optimal MAP (Maximum a Posteriori) approximation MIMO algorithm can be implemented using an iterative technique; however, this incurs high latency penalties. A more practical non-linear MIMO receiver implementation known as ML (Maximum Likelihood) or MLD (Maximum Likelihood Detector) is fundamentally based on an exhaustive constellation search. The MLD is more demanding on processing than a conventional linear receiver, but can offer significantly higher bit-rates for the same channel conditions. In addition, the MLD is more robust to channels with antenna correlation.

Working with high-order MIMO dimensions (more than two receive and two transmit antennas) can result in significantly improved spectral efficiency – but this comes at a cost: the computational complexity of the MLD receiver grows exponentially with the increase of the MIMO dimension. High-order MIMO requires considerable processing power – to the point where a straightforward MLD approach is impractical, and suboptimal MLD algorithms must be used to enable User Equipment (UE) implementations.

This white paper:

- Reviews the relevant MIMO modes and technology.
- Describes the advantages of choosing a suboptimal MLD receiver over a Minimal Mean Square Error (MMSE) receiver.
- Explains the complexities of the MLD implementation and how to solve them using suboptimal ML solutions.

2 MIMO Techniques

MIMO techniques can be split into three main groups:

1. Beam-forming: used to improve the SNR of a given channel
2. Transmit and receive diversity: used to improve channel quality or robustness
3. Spatial Multiplexing: used to increase data throughput for a given channel

Beam-forming utilizes knowledge of the channel at the transmitter to focus the power in the direction of the receiver. Details of the channel can be obtained by receiving feedback from the receiver regarding direction and attenuation properties. By identifying the direction of the UE, the transmitter can steer a beam in that direction, and thus amplify the received signal. This MIMO technique is most effective for low-SNR channels. The figure below describes a directional wave-front achieved by timing the phase of the transmit antennas.

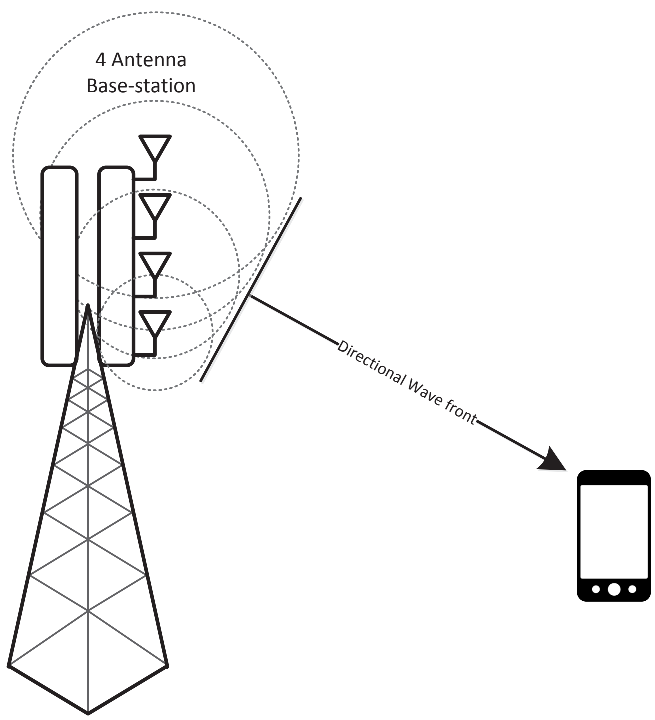

Figure 2-1: Tx Beam-Forming

Transmit and receive diversity creates redundancy by transmitting the same data on multiple antennas, and combining the signals received at the destination antennas to increase the robustness for a given channel. This MIMO technique is most effective for low SNR and rich multipath (or scattering) conditions. The diversity can maximize the utilization of the channel by overcoming attenuations at antennas, and make better use of antennas that receive strong signals. Overall, the SNR obtained at each antenna is improved, and this reduces decoding errors at the receiver.

By improving the SNR, it is possible to increase the throughput by switching to a higher modulation (for example: 64QAM instead of 16QAM/QPSK) or increasing the code rate (transmitting less redundant data). However, improving the SNR of the channel has its limits. It has been shown that above a certain point, the throughput gains for each dB of SNR diminishes rapidly. This ‘knee’ point describes the highest modulation and code rate defined by the standard. In order to further increase the throughput, it is necessary to utilized more advanced transmission methods.

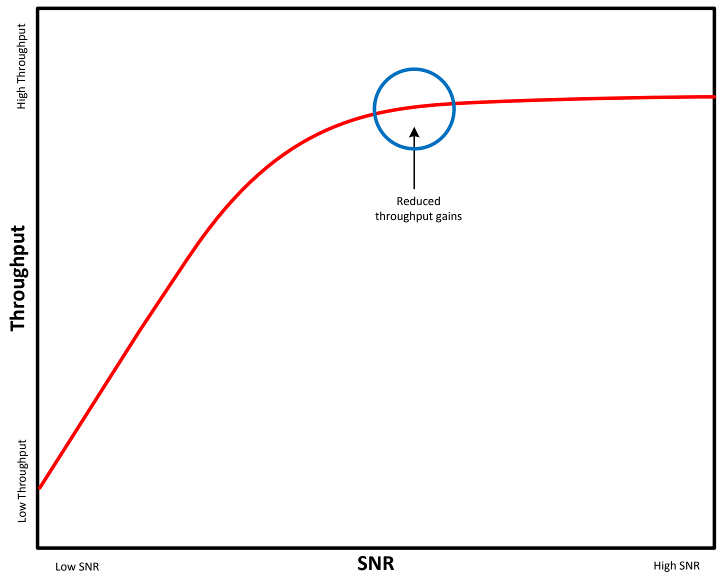

Figure 2-2: STBC Throughput Peak

Spatial multiplexing is introduced in order to push the channel throughput to the next level[i]. Spatial multiplexing requires minimal channel conditions to work effectively. This technique takes advantage of rich multipath channels in order to differentiate between the data transmitted on each antenna – this is called a spatial layer. Rich multipath conditions are generated by the reflections of the transmitted signals from obstacles such as buildings and vehicles in the urban environment. These reflections improve signal separation at the receiver, enabling reconstruction of the data into the layers in which the data was originally transmitted. The number of possible spatial layers is determined by the number of transmitting and receiving antennas. For a configuration of four transmitting antennas and three receiving antennas, the channel can contain up to three spatial layers min (4Tx, 3Rx). The actual number of layers is determined by the multipath conditions denoted as channel rank. For the configuration above (4Tx, 3Rx) with line-of-sight conditions and no multipath reflections, the rank would be equal to one, thereby enabling only a single spatial layer of data. As these conditions improve (multipath increases) we can add more spatial layers – or in other words: multiply the rate of transmitted data on the channel.

There is no single MIMO technique that supports all channel conditions. The eNB (Base station) must adapt the transmission scheme – depending on the multipath, SNR and mobility – many times per second in order to maximize the throughput for each specific UE.

2.1 Antenna Correlation

An important factor in choosing the MIMO technique is the level of antenna correlation. *Figure 2-3* below describes a 2X2 MIMO LTE channel (EPA 5Hz conditions) using two transmission schemes in different antenna correlation propagation conditions:

1. Transmit and Receive Diversity (STBC)
2. Spatial multiplexing (SM)

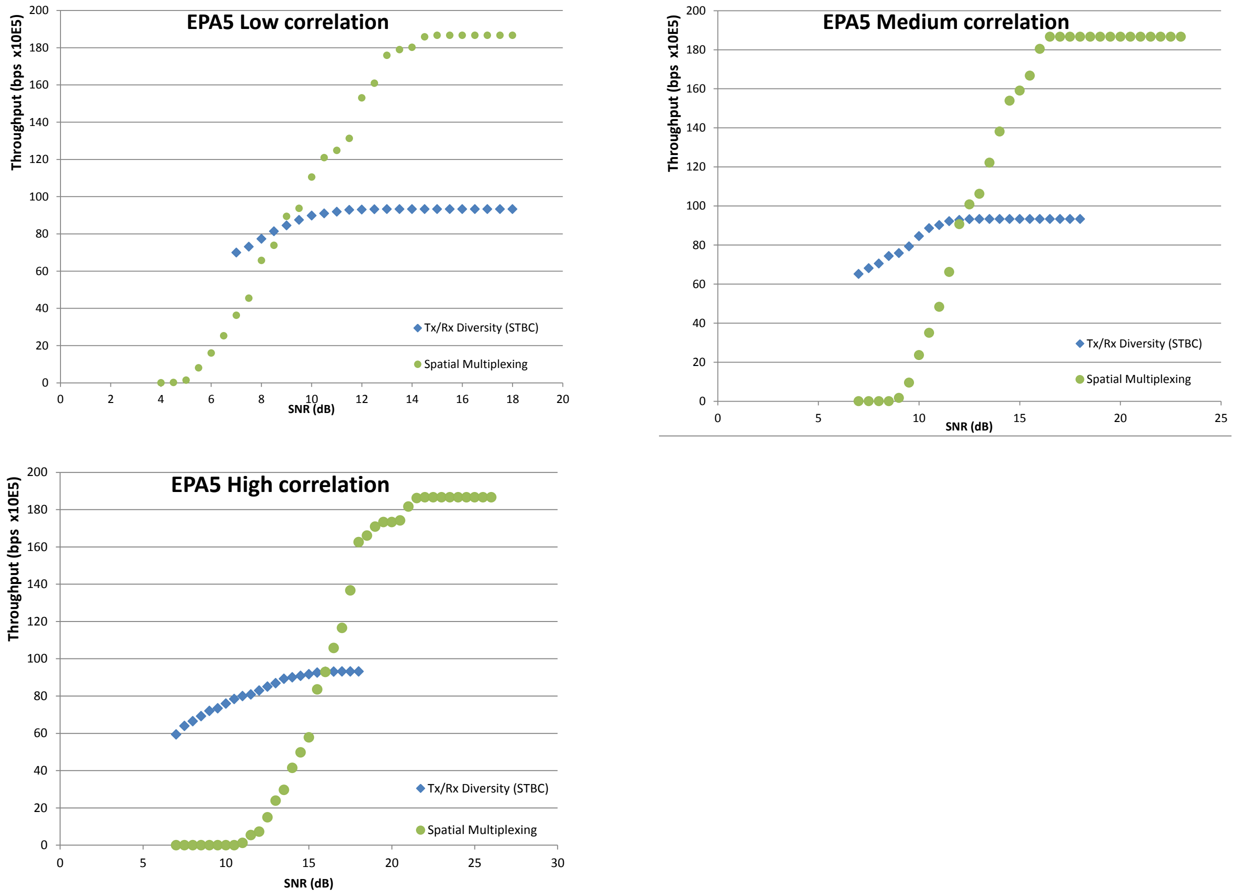

Figure 2-3: Antenna Correlation Effects

It can be observed that for low SNR conditions, STBC provides superior results. In high SNR conditions, SM provides close to twice the throughput delivered by STBC (for 2X2 MIMO). The MIMO order defines the throughput gain at high SNR values – a gain of three for 3X3 MIMO, a gain of four for 4X4 MIMO, and so on. The crossing point between the two graphs represents the SNR value at which SM starts exceeding the STBC throughput. SM is more sensitive to antenna correlation and requires higher SNR values to surpass STBC in high correlation scenarios. Therefore, it is important to choose an SM solution with high immunity to antenna correlation.

3 MIMO Receivers

The UE MIMO receiver has many possible implementations. Among them, the most common are linear receivers, which include the Zero-Forcing (ZF) and Minimal Mean Square Error (MMSE) detectors. Another implementation solution is a non-linear receiver based on the Maximum Likelihood (ML) detector.

Assuming the following mathematical baseband signal model, where:

$$y = Hs + pn$$

- y is the vector of the signals sampled at the receiver. The size of this vector corresponds to the number of receive antennas (Nr).
- s is the vector of transmitted symbols from multiple antennas. The size of this vector corresponds to the number of transmit antennas (Nt).
- H is the channel impulse response matrix that describes the channel between each transmit antenna and to each receive antenna. The dimensions of this matrix correspond to Nr X Nt.
- pn is the vector of independent complex valued Gaussian random variables each with variance of p^2

Receiver performance is evaluated using a tool called an Error Probability Curve.

The Error Probability Curve is a graph that plots the channel SNR on the X-axis and the error rate on the Y-axis. The channel SNR is measured in dB. The error rate is a logarithmic axis that can have several representations – bit error rate (BER), symbol error rate (SER) or packet error rate (PER).

The packet error rate (PER) is the number of incorrectly received data packets divided by the total number of received packets. A packet is declared incorrect if at least one bit is erroneous. For coded communications systems, PER is measured including the FEC (Forward Error Correction) decoder.

Figure 3-1: Diversity Order and Figure 3-2: Array Gain refer to BER – the number of incorrect bits divided by the total amount of transmitted bits.

The Error Probability Curve for MIMO receivers is characterized by two main parameters: diversity order and array gain.

Diversity order (DO) is defined as the slope of error probability curve at high SNR. The greater the DO, the steeper the slope of the Error Probability Curve – higher DO is preferable.

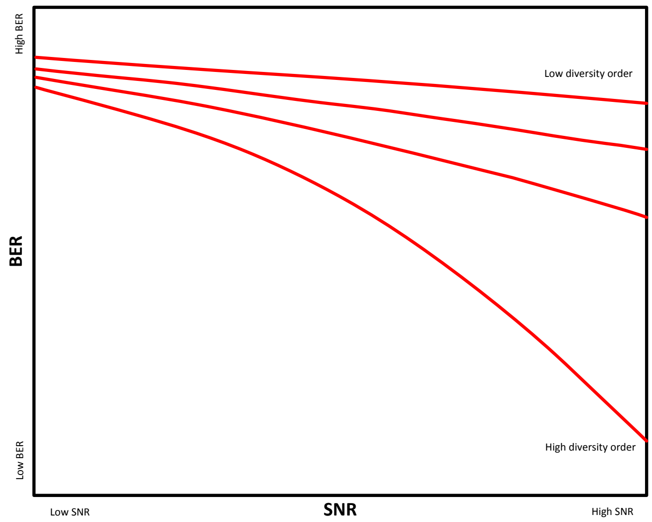

Figure 3-1: Diversity Order

Array Gain (AG) is defined as the horizontal shift of the Error Probability Curve. For a higher AG value, the slope of the Error Probability Curve will be shifted to the left, toward the lower SNR values – in this case, higher AG is preferable.

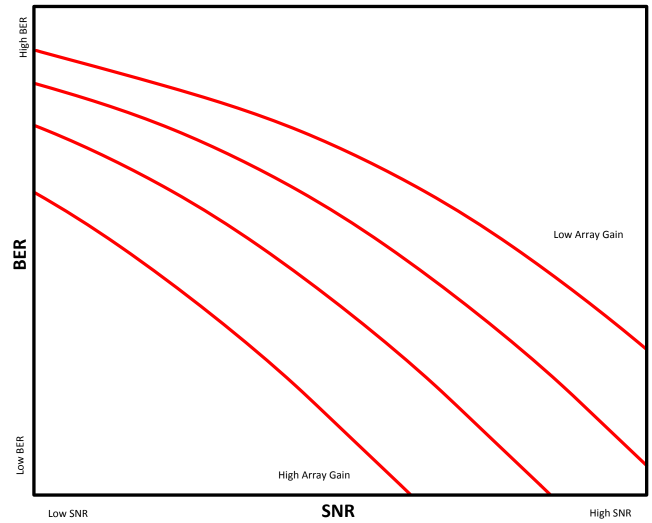

Figure 3-2: Array Gain

Table 3-1 below describes the DO and AG calculations for three types of receivers using spatial multiplexing.

Receiver Type	Receiver Equation	Diversity Order	Array Gain
Zero Forcing (ZF)	$\hat{s}_{ZF} = (H^* H)^{-1} H^* y$	$Nr - Nt + 1$	$\frac{N_r - N_t + 1}{N_t}$
Minimum Mean Square Error (MMSE)	$\hat{s}_{MMSE} = (H^* H + p^2 I)^{-1} H^* y$	$Nr - Nt + 1$	$\frac{N_r - N_t + 1}{N_t}$
Maximum Likelihood (ML)	$\hat{s}_{ML} = argmin \lVert y - Hs \rVert^2$	Nr	$\frac{N_r}{N_t}$

Table 3-1: Spatial Multiplexing Parameters

Using the table above it is simple to calculate a 4X4 MIMO configuration (Nr = 4, Nt = 4) transmitting with spatial multiplexing.

The diversity order for an ML receiver will be 4, equal to the number of receive antennas (Nr = 4) compared to a diversity order of 1 for the linear receivers (Nr - Nt +1 = 4 – 4 + 1 = 1). This indicates a clear advantage of the ML receiver compared to the linear solution – especially at high SNR values.

The same calculation for array gain will produce an array gain of 1 for the ML receiver, compared to an array gain of ¼ for the linear solution. Once again, the ML receiver provides superior results.

It is observable that the main benefits of the ML receivers are at high SNR values. Under these conditions, the DO and AG parameters are significantly greater than the linear receivers. On the one hand, this indicates that for low SNR conditions it might be sufficient to implement a simpler linear receiver or refrain from spatial multiplexing altogether, and choose a more robust transmission scheme. On the other hand, for spatial multiplexing with high throughput at adequate SNR, the ML receiver is the obvious choice.

This paper refers to soft-output MIMO solutions as opposed to hard-output solutions. Instead of producing definitive bit solutions of ‘one’ or ‘zero’, the soft-output solution consists of a ratio between the probability that a certain bit is ‘one’ and the probability that it is ‘zero’. This ratio is denoted as soft-bit or LLR (log likelihood ratio).

3.1 Turbo MIMO

The methods described above are called 'one shot' as they complete the processing of the input signal or tone after a single activation of the detector.

Another approach that aims for MAP performance offers an iterative solution that involves a soft symbol detector and an external FEC decoder[ii].

The FEC decoder is a separate module in the receiver which performs forward error correction (FEC). By taking advantage of redundancy introduced into the transmitted signal the FEC decoder can detect errors in the received bit stream and often correct these errors without the need for retransmission.

A turbo-MIMO receiver structure consists of two stages: soft-output symbol detector and FEC decoding. In the first iteration, the symbol detector produces LLR results based only on the received input signal. The FEC decoder will then weaken or strengthen LLRs, in accordance with the coding constraints. Subsequently, the symbol detector reiterates taking advantage of the prior knowledge of the LLRs supplied by the FEC decoder. These two stages iteratively exchange information transferred from one to the other until the receiver converges.

The symbol detector may consist of a soft-output ML detector implementation, or alternatively may use a simpler zero-forcing/MMSE detector followed by a soft-symbol de-mapper.

By performing this iterative process, the receiver can surpass the precision of the ML-decoder and obtain lower error rates.

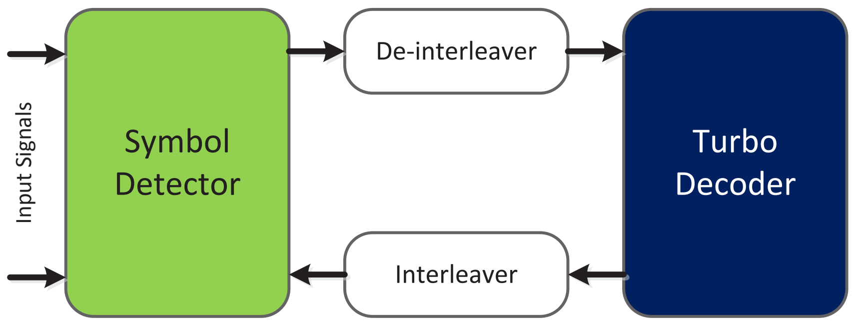

Figure 3-3: Turbo MIMO Receiver

Advantages of this receiver are:

- Very high precision results can be obtained, exceeding the ML solution and approaching Maximum a Posteriori (MAP) results.
- The symbol detector can be simplified to a linear solution at the expense of more iterations between the detector and the turbo decoder.

Disadvantages of this receiver are:

- Large data transfers are required between the FEC decoder and the symbol detector; these need to be scheduled and stored in intermediate buffers.
- Latency is increased due to multiple iterations and transfers.
- Throughput is reduced.
- Additional power consumption due to multiple data transfers and iterations.

4 ML Receiver Implementation

The ML receiver has significant advantages, but these come at a price of implementation complexity. The Maximum Likelihood (ML) Receiver estimator solves the following equation:

$$\tilde{S}_{ML} = argmin \| y - Hs \|^2$$

For the sake of simplicity, let's use a SISO single transmit and receive antenna configuration as an example. In this case, *y* is the signal sampled at the receiver, *s* is the transmitted symbol, and *H* is the channel impulse response describing the channel between the transmit antenna and receive antenna.

The receiver looks for the transmitted symbol *s*, which minimizes this absolute value: $\| y - HS \|$. *s* belongs to a group of finite values that are defined by the symbol modulation. For 64QAM modulation, for example, *s* can have 64 different values.

Basically, this boils down to an exhaustive search. The receiver must scan all possible values of *s* to find the one that when multiplied by the estimated channel *H* will be closest to the received signal.

For a SISO system this is quite simple, but when moving to a MIMO system the complexity grows exponentially. For example, in a 2X2 MIMO configuration with 64QAM modulation, *s* is a vector of two values. The first antenna can transmit 64 different symbols and the second antenna can also independently transmit one of 64 possible symbols. There are a total of 64^2 or 4096 values of *s* that must be evaluated.

For 2X2 MIMO, a number of algorithms are used to reduce complexity of the ML receiver[iii]. Worth noting is the LORD algorithm[iv], which is capable of reducing the search complexity from 64^2 options to 64*2 or 128 evaluations reaching ML precision.

For 4X4 MIMO 64QAM this number now grows to 64^4 – or 16,777,216 different values of *s* that must be evaluated. Solving this magnitude of complexity requires a new approach; this is where the suboptimal ML receivers come in to play.

4.1 Suboptimal ML Receivers

Suboptimal ML receivers try to scan the possible transmitted signals in a more efficient way, thereby reducing the overall complexity and reaching near-ML precision results. The reduced complexity contributes to a more practical hardware implementation in terms of area and power. This also enables the hardware to keep up with the high throughputs defined by advanced communications standards.

Solving a suboptimal ML equation may be defined as a tree search[v],[vi],[vii], in which each level of the tree corresponds to a transmitted symbol. The number of the branches protruding from each node matches the QAM or modulation of the transmitted symbol. A 4X4 MIMO configuration is represented by a four-level tree. If the modulation is BPSK, each node will contain two branches.

Once the tree symbol is defined, tree traversal algorithms may be deployed, borrowing from other fields such as computer science.

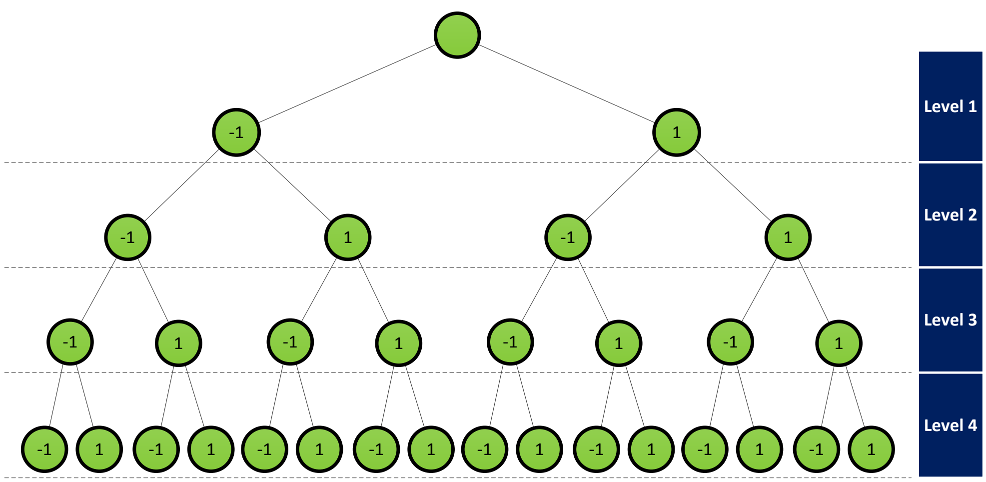

Figure 4-1: MIMO Symbol Tree

In this context, suboptimal ML receivers can be partitioned into two main types:

- Breadth first search
- Depth first search

4.1.1 Breadth First Search

An example of breadth first is the K-best algorithm[i,viii,ix]. This decoder is a fixed-complexity solution that starts from the tree root and ascends until it reaches the last level of the tree. At each level of the tree, all selected branches are evaluated and K survival nodes are preserved, matching the best solution (representing the symbols closest to the received signals) – hence the name 'K-best'. The K remaining leaves are then used to generate the LLR results.

Advantages of this decoder are:

- Unidirectional flow contributes to the easy pipelining implementation in hardware.
- Processing power required to calculate each level is constant, and directly related to the number of survival nodes (K) selected in the implementation.
- Throughput is constant, which in turns simplifies data flow scheduling in the system.

Disadvantages of this decoder include:

- Large area implementation is required in order to evaluate and sort all the selected nodes of the level.
- The larger the precision requirements, the higher the K value required.
- Throughput does not increase in optimal SNR conditions.
- Reaching the ML solution is not guaranteed, because the best solution might reside in the nodes that are not selected.

The following figure shows a MIMO 4X4 (4-level) tree with QPSK modulation. K in this case is four. Sixteen nodes will be sorted at each level of the tree. The best four will be the surviving nodes for the next level.

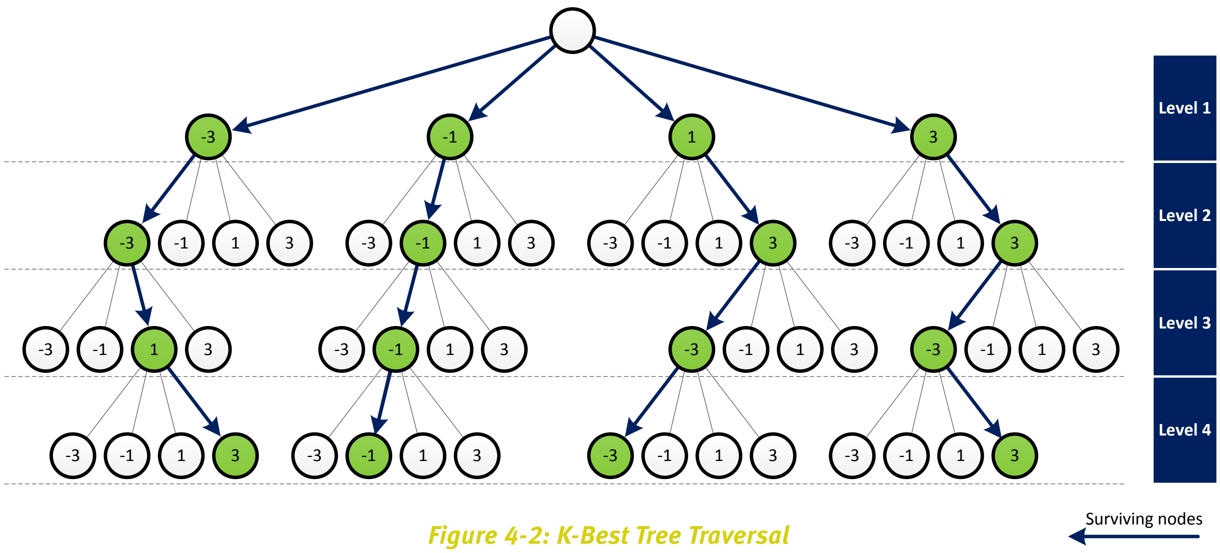

Figure 4-2: K-Best Tree Traversal

4.1.2 Depth First Search

An example of depth first is the Soft-Output Sphere Decoder algorithm[x],[xii]. This decoder is an adaptive complexity solution that starts from the tree root and primarily ascends directly to a tree leaf – hence the name 'Depth First'. This first solution of the tree determines an initial search radius or sphere. From then on, the decoder backtracks and ascends throughout the levels of the tree. Each node of the tree that exceeds the search radius is pruned together with all the nodes underneath it. Each time a better solution is found, the radius is reduced accordingly. In this way, the symbol tree is scanned and pruned until the number of valid options is reduced. The remaining symbols represent the ML solution.

Advantages of this decoder are:

- Obtaining the ML solution is guaranteed, contributing to the precision of the result.
- Under high SNR conditions the decoder performs faster, increasing throughput and reducing power consumption.
- Smaller area implementation compared to equivalent breadth first solution.

Figure 4-3 shows a cycle count comparison between a Soft-Output Sphere decoder with adaptive complexity compared to a fixed complexity K-best decoder. As the SNR increases, the sphere decoder will reduce its cycle count while the fixed complexity will stay constant, regardless of the channel conditions.

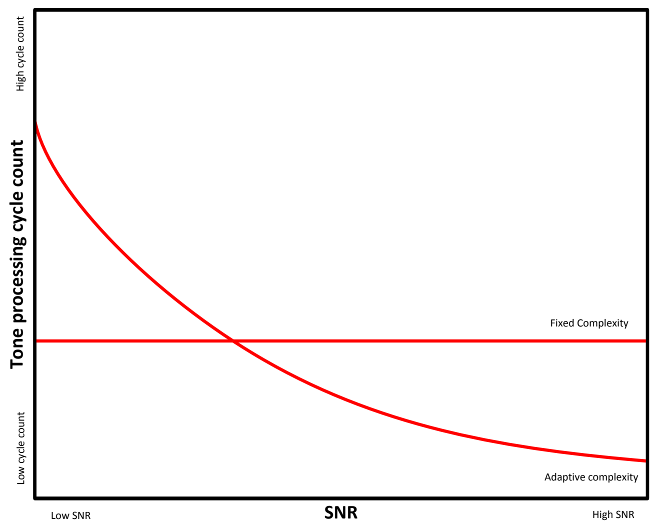

Figure 4-3: Fixed vs. Adaptive Complexity

Disadvantages of this decoder include:

- Non-deterministic behavior of the decoder complicates system scheduling.
- Next branch selection is known only after the current branch is complete. This makes the hardware pipeline implementation challenging.

Figure 4-4 shows an example MIMO 4X4 (4 level) tree with QPSK modulation.

1. Depth first chooses the symbol path to the first leaf in the following manner:
    a. -3 (level 1)
    b. -3 (level 2)
    c. 1 (level 3)
    d. 3 (level 4)
2. Initial Radius is updated.
3. Backtrack is performed to a symbol at level 2.
4. Branches that exceed the search radius (shown in red) are pruned during the search, thereby minimizing the search tree.

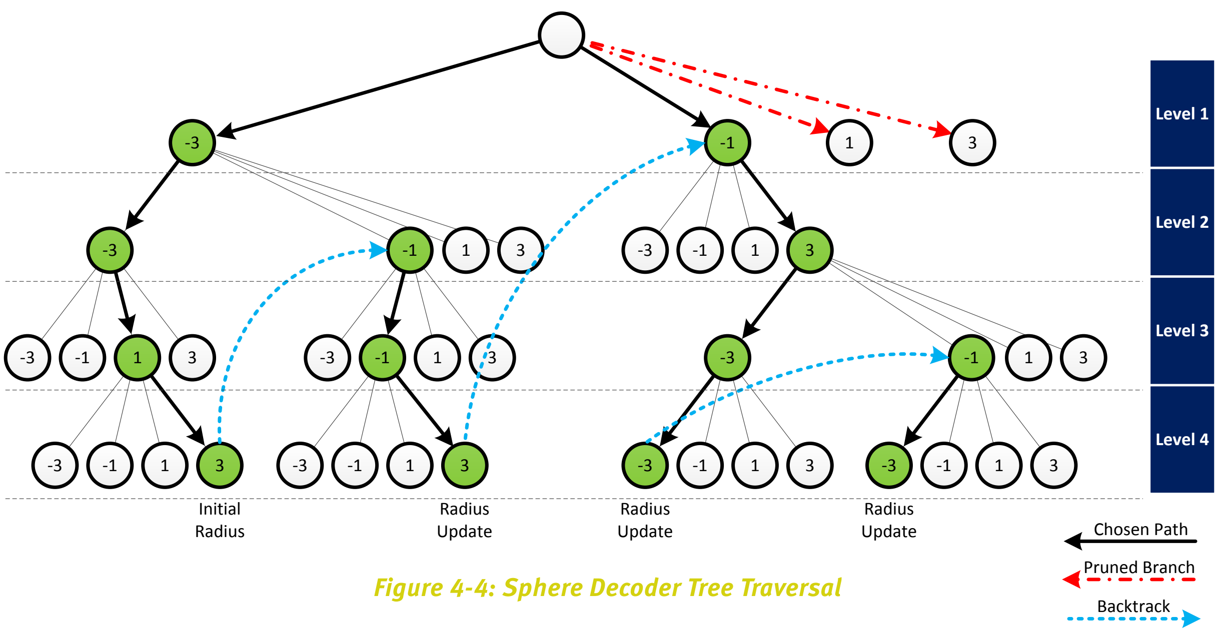

Figure 4-4: Sphere Decoder Tree Traversal

5 CEVA Solution

CEVA responds to the challenges of the MIMO receiver by introducing the Maximum Likelihood MIMO Detector (MLD). The MLD is a Tightly Coupled Extension (TCE) accelerator hardware unit. The MLD is capable of processing LTE – Advanced Cat.7 data streams and produces soft-output max-log ML solutions.

The MLD accelerator reaches suboptimal Maximum Likelihood (ML) solution for 4X4 or 3X3 MIMO @12.6 Mega-tones/sec using a Soft-Output Sphere Decoder approach and 2X2 LORD based ML solution @28.8 Mega-tones/sec using carrier aggregation. The MLD is designed for mobile applications, emphasizing a low-power design concept.

5.1 Feature Set

The MLD feature set includes support for:

- Variable transmission schemes from 2X2 up to 4X4 MIMO, with configurable modulation per layer of up to 64QAM.
- Tree search optimization: user-defined layer ordering, initial radius and search radius for each tree level.
- CEVA MLD addresses the non-deterministic nature of the Soft-Output Sphere Decoder by presenting throughput control capabilities, including lower and upper cycle count boundaries for tone processing. In addition, the system throughput is maintained using user-defined timestamp-based termination.
- Soft-bits can be scaled to account for SNR and modulation factors.
- Support is provided for LLR permutations at intra-symbol and inter-layer resolutions.
- Internal layer de-mapping: two code layers are supported, enabling the MLD to split the written data to two different destinations.
- Scalable hardware solution enables Performance/Power/Area trade-offs, including choosing the number of MLD engines, buffers sizes and interface clock ratio.

In addition, the accelerator provides extensive debug and profiling capabilities.

5.2 MLD Accelerator Block Diagram

Figure 5-1 describes a block diagram of the MLD accelerator which consists of an AXI interface, Input Buffer, Dispatcher, MLE (Maximum Likelihood Engine), LLR Generator, Reorder Buffer and Output Buffer.

The Input buffer stores a multitude of tone data that is transferred one tone at a time via the dispatcher to the MLEs. Each MLE outputs data regarding the detected bits; this in turn is transformed into LLR format by the LLR Generator. The Reorder Buffer accumulates the LLR data in order of transmission and sends the organized output toward the Output Buffer. The Output Buffer writes the LLRs to the next block in the receive chain via the AXI interface.

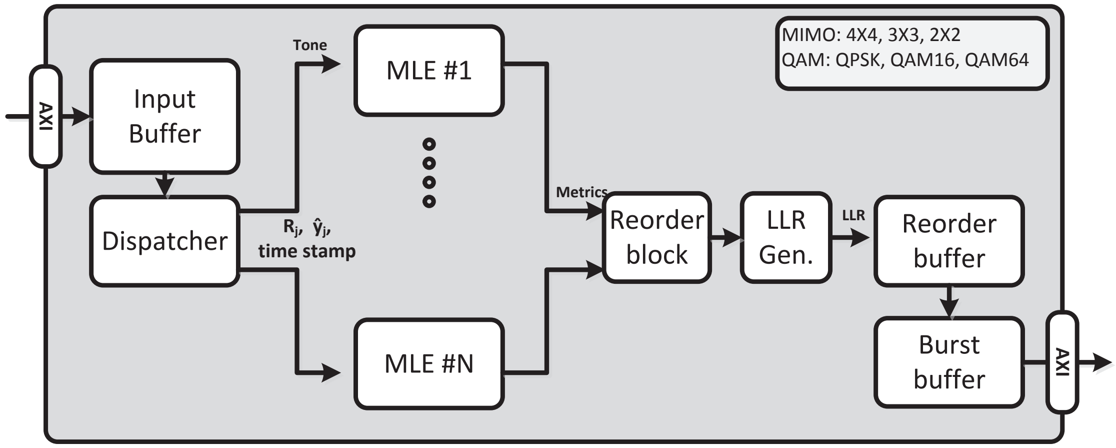

Figure 5-1: MLD Accelerator Block Diagram

5.3 MLD Performance

Figure 5-2 describes CEVA MLD TCE performance compared to an MMSE receiver using 4X4 Spatial Multiplexing MIMO. Throughput in PER (Packet Error Rate) is evaluated at different SNR conditions. The LTE channel is set at EPA 5Hz with low correlation propagation conditions.

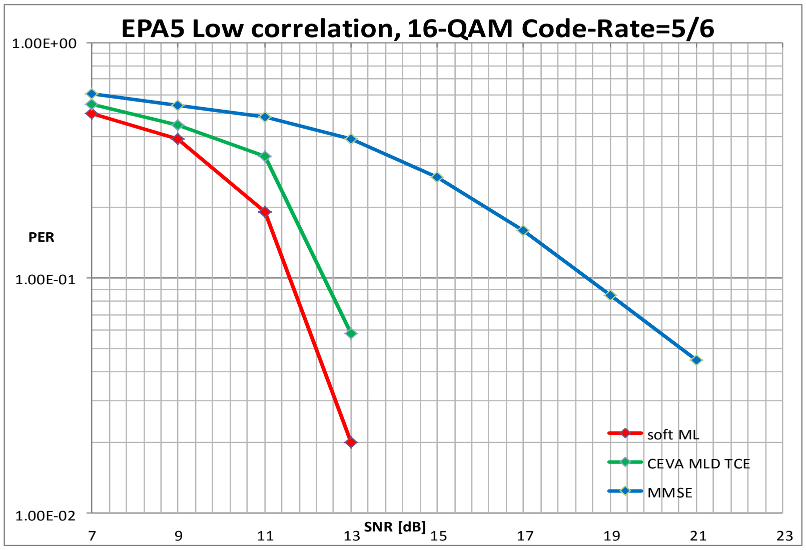

Figure 5-2: 4X4 MIMO Spatial Multiplexing Performance

CEVA's solution obtains near-ML results, while MMSE suffers from severe performance degradation even in low correlation conditions. For higher correlation conditions, the MMSE will worsen even further.

For comparison, a K-best solution with similar performance will require more than twice the area of the CEVA MLD TCE.

CEVA MLD TCE boasts:

- Superb precision of less than 1.5dB loss for MIMO 4x4 compared to pure ML decoding.
- Decoding MIMO 2x2 with no precision loss (LORD equivalent performance and complexity).
- Ultra-low-power design.
- Competitive die size.

Figure 5-3 describes the performance of 4X4 MIMO with SM at peak code rate with 64-QAM modulation. Even in these conditions, the CEVA MLD TCE provides less than 1.5dB loss compared to ideal ML results.

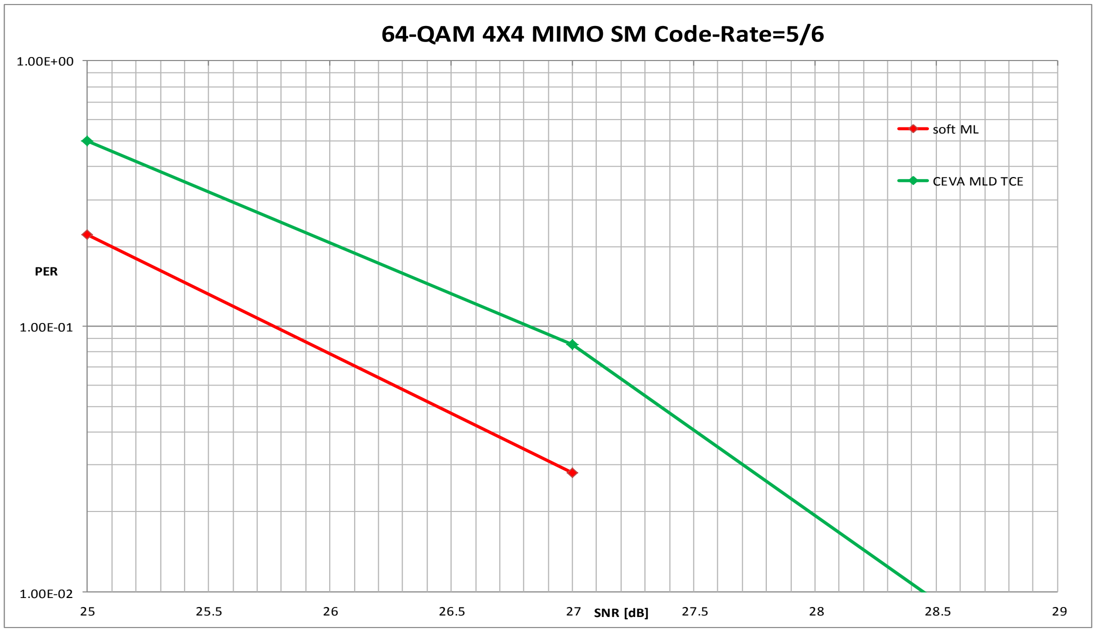

Figure 5-3: MLD 4X4 MIMO Performance

Figure 5-4 illustrates the performance of 2X2 MIMO with SM at peak code rate with 64-QAM modulation. The CEVA MLD TCE provides perfect ML performance.

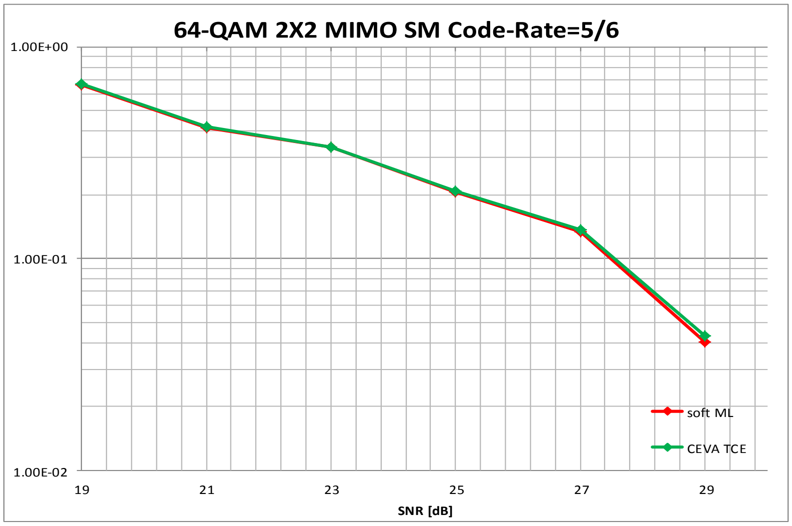

Figure 5-4: MLD 2X2 MIMO Performance

6 Conclusion

MIMO is a key component of next-generation wireless technology; in order to fully exploit the potential data rate, it is essential to deploy spatial multiplexing techniques.

This paper has shown that the MLD receiver achieves superior results to the linear receiver but there are many factors that need to be considered when choosing an MLD implementation.

The MLD receiver designer must choose the most appropriate solution to the required application, taking the following into account:

- Precision targets and throughput requirements: demands a user-configurable solution in order to obtain high-quality LLRs quickly
- Latency definitions: calls for definable system scheduling in order to complete the task in the allotted time – for example, by using timestamps
- Channel type fast/slow time variant: a fast time variant channel will require the ability to frequently update channel information
- Hardware considerations: Area, Speed (MHz) and Power dissipation
- Requires a scalable hardware solution to meet small area and low power requirements

Choosing an optimized MLD receiver can be the main differentiator in a cellular product.

7 References

[i] P.W. Wolniansky, G.J. Foschini, G.D. Golden and R.A. Valenzuela, "V-blast: an architecture for realizing very high data rates over the rich-scattering wireless channel", Signals, Systems, and Electronics, URSI International Symposium on, vol. 1998, pp. 295-300, 1998.

[ii] B. M. Hochwald and S. ten Brink, "Achieving near-capacity on a multiple-antenna channel".

[iii] Y. Lomnitz and D. Andelman, "Efficient maximum likelihood detector for MIMO systems with small number of streams", Electronics Letters, vol. 43, no. 22, pp. 1212–1214, 2007.

[iv] M. Siti, and M.P. Fitz, "A Novel Soft-Output Layered Orthogonal Lattice Detector for Multiple Antenna Communications", IEEE International Conference Communications, 2006. ICC '06.

[v] U. Fincke and M. Pohst, "Improved methods for calculating vectors of short length in a lattice, including a complexity analysis", *Math. of Comput.*, vol. 44, pp. 463-471, Apr. 1985.

[vi] B. Hassibi and H. Vikalo, "On the expected complexity of sphere decoding," in Proc. 35th Asilomar Corf. Signals, Syst., Comput., vol. 2, 2001, pp. 1051-1055.

[vii] B. Mennenga, A. von Borany, and G. Fettweis, "Complexity reduced soft-in soft-out sphere detection based on search tuples", *Proc. of the IEEE Int. Conf. Commun.*, Dresden, Germany, pp. 1-6, Jun. 2009.

[viii] R. Shariat-Yazdi and T. Kwasniewski, "Reconfigurable K-best MIMO detector architecture and FPGA implementation," International Symposium on Intelligent Signal Processing and Communication Systems (ISPACS), pp. 349-352, Dec. 2007.

[ix] K.Wong, C. Ysui, S. Cheng and W. Mow, "A VLSI architecture of a K-Best lattice decoding algorithm for MIMO channels", *IEEE Trans. Circuits Systems*, vol. 3, pp. 273-276, May 2002.

[x] C. Studer, A. Burg, and H. Bolcskei, "Soft-output sphere decoding: Algorithms and VLSI Implementation", *IEEE J. Select. Areas in Commun.*, vol. 26, no. 2, pp. 290-300, Feb. 2008.